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LETTER TO THE EDITOR

Critical exponents of self-avoiding Levy flights

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Abstract. Monte Carlo simulations are presented for one-dimensional self-avoiding Levy flights. They are in perfect agreement with the predictions of Flory theory. Some field-theoretic predictions are also verified, although much less clearly. In particular, ε expansions are available but their range of applicability seems to be small.

Critical phenomena in systems with power-behaved long-range forces have been studied since the early days of the renormalisation group [1, 2]. But only recently have the simplest such systems, namely self-avoiding Levy flights, been simulated by Monte Carlo techniques in order to verify these predictions [3].

Levy flights are similar to random walks, except that the steps are not necessarily to next neighbours [4]. Instead, the probability for a step to have a length greater than some r is assumed to decrease like $r^{-\sigma}$ with $0<\sigma<2$, for $r\to\infty$. In analogy to the relation between self-avoiding walks and spin systems found by de Gennes [5], the critical behaviour of self-avoiding Levy flights ('Levy-SAWS') is described by the n-vector model with $n=0$ and with a potential decreasing $\sim r^{-\sigma-d}$ (d is the dimensionality of space) [3].

Starting from this latter connection, critical exponents can be calculated for $d\approx 2\sigma$, using expansions in $\varepsilon=2\sigma-d$ up to second order [1, 2]. We should mention that $d=2\sigma$ is the upper critical dimension. Above d_{crit}, one has $\nu=1/\sigma$ and $\gamma=1$. Here, ν is defined via the geometric average of the end-to-end distance of N-step flights, by $\langle\log R_N\rangle\sim\nu\log N$, and γ is defined via the survival probability of an N-step flight as

$$p_N\sim e^{-\mu N}N^{\gamma-1}. \tag{1}$$

The most important result of references [1, 2] is that the exponent η, related to ν and γ by the standard scaling relation $\gamma=(2-\eta)\nu$, is unrenormalised to all orders of perturbation theory. Since its bare value is $\eta=2-\sigma$, we would predict $\gamma=\nu\sigma$.

In a subsequent paper by Sak [6], it was shown that this can only be true as long as σ is smaller than $2-\eta_{\text{SAW}}$. Above that value, he argued that $\eta=\eta_{\text{SAW}}$. For one-dimensional ordinary SAWS, one has $\gamma=\nu=1$. Thus, one has in one dimension $\eta_{\text{SAW}}=1$. Taking these two predictions together, we obtain therefore for Levy-SAWS in one dimension

$$\gamma=\begin{cases}\nu\sigma & \text{for } \sigma<1\\ \nu & \text{for } \sigma>1.\end{cases} \tag{2}$$

Notice that the claim of reference [3] that the critical exponents are the same as in ordinary SAWS if $\sigma>2-\eta_{\text{SAW}}$ (see also figure 3 of [3]) is wrong.

In addition to this, one has the ε expansions of references [1, 2]. For fixed d these authors obtained

$$\nu=\frac{2}{d}\left(1-\frac{\varepsilon}{2d}+\frac{\varepsilon^2}{d^2}\frac{304-5d^2}{256}\pm\ldots\right) \tag{3}$$

which corresponds to the following expansion for fixed σ:

$$\nu=\frac{1}{\sigma}\left(1+\frac{\varepsilon}{4\sigma}+\frac{\varepsilon^2}{64\sigma^2}(19-\tfrac{5}{4}\sigma^2)\pm\ldots\right). \tag{4}$$

In order to test these predictions, I have performed Monte Carlo simulations in one dimension. The probabilities of making steps from x to $x\pm r$ ($r=1,2,\ldots$) were chosen as $(r^{-\sigma}-(r+1)^{-\sigma})/2$. The flights stopped whenever a step ended at an already visited site. In order to overcome this attrition, I used a standard enrichment method [7]. Typically, the longest flights had been 50 and 100 steps. The number of such flights for each value of σ was $\sim 3\times10^4$–10^5. This is $\sim$1–2 orders of magnitude higher statistics than in reference [3]. The CPU time for each value of σ was $\sim$2–3 h on a CYBER 170/720.

In figure 1, I show the quantity

$$\nu_N=\frac{N\exp\langle\log R_N\rangle}{\tfrac{1}{2}[\exp\langle\log R_1\rangle+\exp\langle\log R_N\rangle]+\Sigma_{n=2}^{N-1}\exp\langle\log R_n\rangle}-1 \tag{5}$$

plotted against $1/N$. We would expect that $\nu_N\to\nu$ for $N\to\infty$, with corrections $\sim 1/N$ if the corrections to scaling have exponent >1, with logarithmic corrections for $\sigma=1/2$ and $\sigma=2$, and with corrections $\sim N^{-\Delta}$ with $0<\Delta<1$ elsewhere. I use this version of the ratio method instead of the more conventional log–log plots since (i) one can better read off error estimates, and (ii) corrections to scaling can easily be overlooked in log–log plots.

The extrapolated values of ν are shown in figure 2. Also shown in that figure are the predictions of equations (3) (broken curve) and (4) (chain curve). We see that these expansions have very little predictive power, unless they could be combined with some other (non-perturbative) result.

In contrast to this, our results serve as a very stringent check of Flory theory. Following Flory's theory for ordinary SAWs [8, 9], we estimate the potential energy of an N-link chain with radius R as

$$E_{\mathrm{pot}}\propto N^2/R^d \tag{6}$$

and its entropy as

$$S\propto R^\sigma/N. \tag{7}$$

Minimising the free energy then yields $R\sim N^\nu$ with

$$\nu=\frac{3}{\sigma+d}. \tag{8}$$

For $d=2\sigma$, this gives $\nu=1/\sigma$ in agreement with field theory.

For $\sigma\approx 1/2$, Flory theory also seems to agree with the data, although it disagrees there with the ε expansions, but the Monte Carlo data are not really precise enough to decide between them on the one hand and Flory theory on the other.

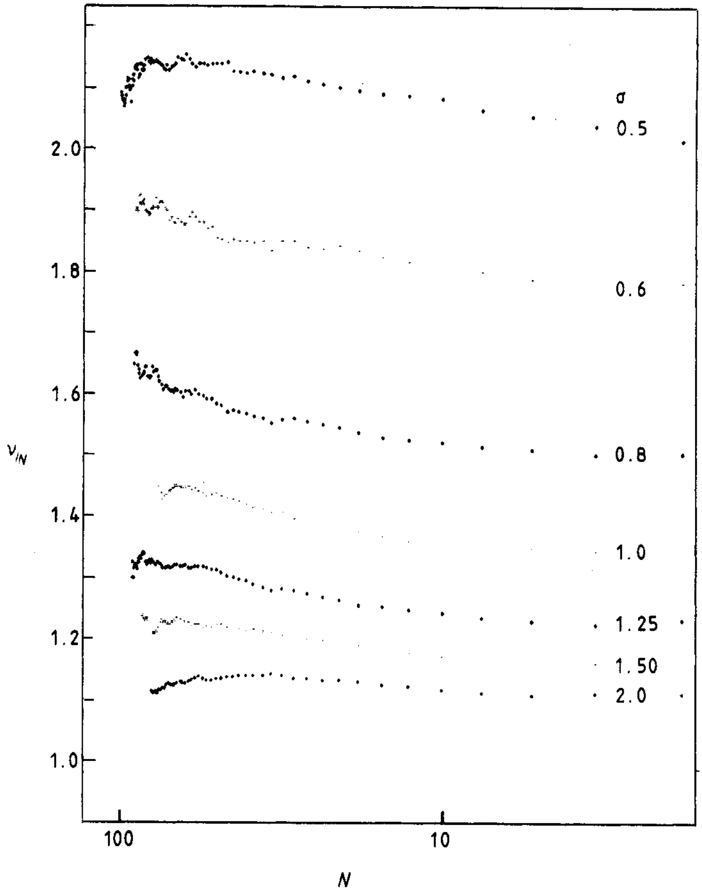

Figure 1. The quantity ν_N defined in equation (5), plotted against $1/N$. The extrapolation towards $1/N \rightarrow 0$ gives the critical exponent ν.

Finally, the Monte Carlo data were also used to estimate the exponent γ. From equation (1), one finds that

$$A_N \underset{\text{def}}{=} \log \frac{p_N}{p_{N-1}} \approx \mu + \frac{\gamma - 1}{N}. \tag{9}$$

Values of A_N for $\sigma = 1.5$ are plotted against $1/N$ in figure 3. From equation (9), we see that the slope of A_N at $1/N = 0$ is just $\gamma - 1$. The resulting value of γ is plotted in figure 4, together with those obtained for other values of σ. In this figure, the predictions of the ε expansion are also plotted. Again the comparison is inconclusive.

Also shown in figure 4 are the predictions of equation (2). We see that they are clearly supported. In particular, the alternative prediction of reference [10] is clearly ruled out.

Summarising we can say that some of the field theoretic results for the problem at hand have rather poor predictive power. This is in contrast to Flory theory which makes simple, clear-cut, and at least approximately correct predictions—in spite of its

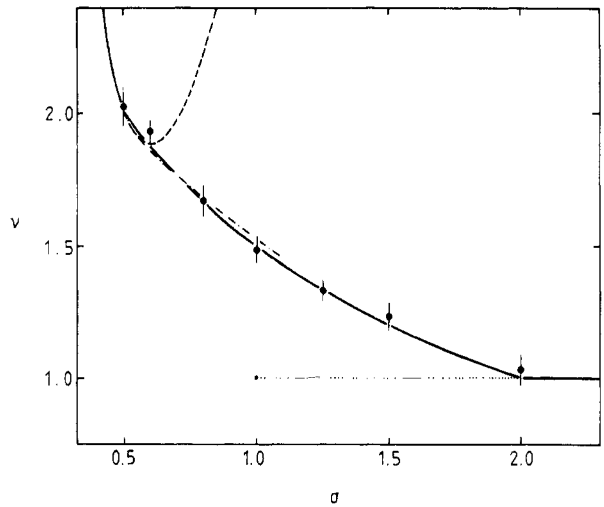

Figure 2. Monte Carlo estimates of ν obtained from extrapolations in figure 1 (dots with error bars), compared to various theoretical predictions. Full curve: Flory theory, equation (8); broken curve: ε expansion, equation (3); chain curve: ε expansion, equation (4).

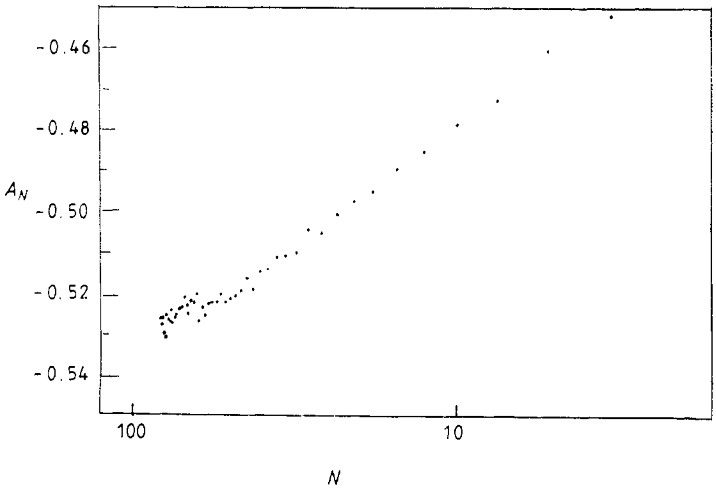

Figure 3. The quantity A_N defined in equation (9) for $\sigma = 1.5$ plotted against $1/N$. The slope at $1/N \to 0$ gives $\gamma - 1$.

inherent flaws [9]. The only field theoretic prediction which is clearly supported by the data is equation (2).

These results leave open the question whether models with power-behaved long-range interactions can in general serve as tests of ε expansions [3]. This would be very useful, as other models (ordinary and directed percolation) with long-range interactions can also be formulated as interacting Levy flights, and could be studied by similar Monte Carlo methods. The upper critical dimension for long-range percolation is $d_{\text{crit}} = 3\sigma$, while for directed percolation it is 2σ.

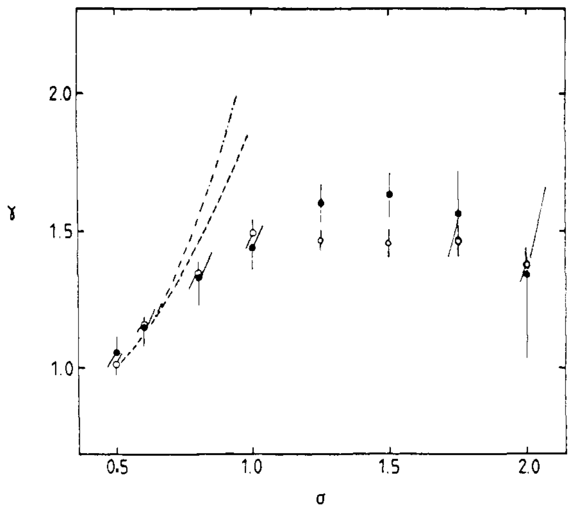

Figure 4. Critical exponent γ plotted against σ. Full points: Monte Carlo estimates from figure 3; broken curve: ε expansion, equation (7), reference [1]; chain curve: ε expansion, equation (9), reference [1]; open points: predictions of equation (2).

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